B.A/ B.Sc	Semester-IV	Credits:4
Course:4	MATHEMATICS REAL ANALYSIS	Hrs/Weak:5

Course Outcomes:

After successful completion of this course, the student will be able to

- get clear idea about the real numbers and real valued functions.
- obtain the skills of analyzing the concepts and applying appropriate methods for testing convergence of a sequence/ series.
- Test the continuity and differentiability and Riemann integration of a function.
- Know the geometrical interpretation of mean value theorems.

UNIT I: **(12 Hours)**

Introduction of Real Numbers (No question is to be set from this portion)

Real Sequences: Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence. The Cauchy's criterion, properly divergent sequences, Monotone sequences, Necessary and Sufficient condition for Convergence of Monotone Sequence, Limit Point of Sequence, Subsequences , Cauchy Sequences – Cauchy's general principle of convergence theorem.

UNIT II: **(12 Hours)**

INFINITIE SERIES :

Series : Introduction to series, convergence of series. Cauchy's general principle of convergence for series tests for convergence of series, Series of Non-Negative Terms.

1. P-test
2. Cauchy's n^{th} root test or Root Test.
3. D'-Alemberts' Test or Ratio Test.
4. Alternating Series – Leibnitz Test.

UNIT III: **(12 Hours)**

CONTINUITY:

Limits: Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limit concept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).

Continuous functions: Continuous functions, Combinations of continuous functions, Continuous Functions on interval.

UNIT IV: **(12 Hours)**

DIFFERENTIATION AND MEAN VALUE THEOREMS: The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean value Theorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

UNIT V: **(12 Hours)**

RIEMANN INTEGRATION : Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Properties of integrable functions, Fundamental theorem of integral calculus, First mean value Theorem.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

TEXT BOOK:

1. Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, published by John Wiley.

REFERENCE BOOKS:

1. A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand & Company Pvt. Ltd., New Delhi.
2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania, published by S. Chand & Company Pvt. Ltd., New Delhi.

B.A/ B.Sc	Semester-IV	Credits:4
Course:5	LINEAR ALGEBRA	Hrs/Weak:5

Course Outcomes:
After successful completion of this course, the student will be able to;

- understand the concepts of vector spaces, subspaces, basises, dimension and their properties.
- understand the concepts of linear transformations and their properties
- apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods
- Learn the properties of inner product spaces and determine orthogonality in inner product spaces.

UNIT I: **(12 Hours)**
Vector Spaces-I: Vector Spaces, General properties of vector spaces, n-dimensional Vectors, addition and scalar multiplication of Vectors, internal and external composition, Null space, Vector subspaces, Algebra of subspaces, Linear Sum of two subspaces, linear combination of Vectors, Linear span Linear independence and Linear dependence of Vectors.

UNIT II: **(12 Hours)**
Vector Spaces-II: Basis of Vector space, Finite dimensional Vector spaces, basis extension, co-ordinates, Dimension of a Vector space, Dimension of a subspace, Quotient space and Dimension of Quotient space.

UNIT III: **(12 Hours)**
Linear Transformations: Linear transformations, linear operators, Properties of L.T, sum and product of LTs, Range and null space of linear transformation, Rank and Nullity of linear transformations – Rank – Nullity Theorem.

UNIT IV: **(12 Hours)**
Matrix : Linear Equations, Characteristic equations, Characteristic Values & Vectors of square matrix, Cayley – Hamilton Theorem.

UNIT V: **(12 Hours)**
Inner product space : Inner product spaces, Euclidean and unitary spaces, Norm or length of a Vector, Schwartz inequality, Triangle Inequality, Parallelogram law, Orthogonality, Orthonormal set, Gram– Schmidt orthogonalisation process. Bessel's inequality and Parseval's Identity.

Co-Curricular Activities **(15 Hours)**
Seminar/ Quiz/ Assignments/ Linear algebra and its applications / Problem Solving.

TEXT BOOK:

1. Linear Algebra by J.N. Sharma and A.R. Vasista, published by Krishna Prakashan Mandir,Meerut- 250002.

REFERENCE BOOKS:

2. Matrices by Shanti Narayana, published by S.Chand Publications.
3. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson Education (low priced edition), New Delhi.
4. Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt. Ltd. 4th Edition, 2007.

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-IV, REAL ANALYSIS

Unit	TOPIC	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Real Sequence	1	2	25
II	Infinite Series	2	2	30
III	Limits and Continuity	1	2	25
IV	Differentiation and Mean Value Theorem	2	2	30
V	Riemann Integration	2	2	30
	TOTAL	8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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Total Marks = 75 M

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MODEL QUESTION PAPER (Sem-End)

B.A./B.Sc. DEGREE EXAMINATIONS

Course-4: REAL ANALYSIS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Prove that every convergent sequence is bounded.
2. Examine the convergence of $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \cdots$
3. Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$.
4. Examine for continuity of the function f defined by $f(x) = |x| + |x-1|$ at x=0 and 1.
5. Show that $f(x) = x\sin\frac{1}{x},\ x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at x=0.
6. Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on $[1,3]$.
7. If $f(x) = x^2\ \forall\, x \in [0,1]$ and $p = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ then find L(p, f) and U(p, f).
8. Prove that if $f: [a,b] \to R$ is continuous on [a, b] then f is R- integrable on [a, b].

SECTION –B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) If $s_n = 1 + \frac{1}{2!} + \frac{1}{3!} + - - - - - - - + \frac{1}{n!}$ then show that $\{ s_n \}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10. (a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n + a^n}$ $(x > 0, a > 0)$

11. (a) Let $f: R \to R$ be such that

$$f(x) = \frac{\sin(a+1)x + \sin x}{x} \quad \text{for } x < 0$$
$$= c \qquad \text{for } x = 0$$
$$= \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} \quad \text{for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

(OR)

(b) If f: [a, b] → R is continuous on [a, b] then prove that f is bounded on [a, b]

12. (a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \forall x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem...

13. (a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-V, LINEAR ALGEBRA

Unit	TOPIC	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Vector spaces - I	2	2	30
II	Vector spaces - II	1	2	25
III	Linear Transformation	2	2	30
IV	Matrices	1	2	25
V	Inner product spaces	2	2	30
Total		8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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Total Marks = 75 M

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MODEL QUESTION PAPER (Sem-End)

B.A./B.Sc. DEGREE EXAMINATIONS

Semester -IV

Course-5: LINEAR ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5M=25 M**

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz=0 is a vector subspace of $V_3(R)$.
2. Define linearly independent &linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of V(R) then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly independent.
3. Prove that every set of $(n+1)$ or more vectors in an n dimensional vector space is linearly dependent.
4. The mapping T : V3(R) → V3(R) is defined by T(x,y,z) = (x-y,x-z). Show that T is a linear ransformation.
5. Let $T: R^3 \rightarrow R^2$ and $H: R^3 \rightarrow R^2$ be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.
6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.
7. State and prove parallelogram law in an inner product space V(F).
8. Prove that the set S = $\left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the Necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$

(OR)

(b) Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of $V_3(C)$ form linearly dependent set, but any three of them are linearly independent.

10. (a) Define dimension of a finite dimensional vector space. If W is a subspace of a finite Dimensional vector space V (F) then prove that W is finite dimensional and dim W $\leq n$.

(OR)

(b) If W be a subspace of a finite dimensional vector space V(F) then Prove that

. $\dim V/W = \dim V - \dim W$

11. (a) Find T (x, y, z) where $T: R^3 \rightarrow R$ is defined by T (1, 1, 1) =3, T (0, 1, -2) =1,T (0, 01) = -2

(OR)

(b) State and prove Rank Nullity theorem.

12. (a) Find the eigen values and the corresponding eigen vectors of the matrix A= $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

(OR)

(b) State and prove Cayley-Hamilton theorem.

13. (a) State and prove Schwarz's inequality in an Inner product space V(F).

(OR)

(b) Given $\{(2,1,3),(1,2,3),(1,1,1)\}$ is a basis of R^3(R).Construct an orthonormal basis using Gram-Schmidorthogonalisation process.
